Understanding the Vocabulary used in Probability

a) An **experiment** is a repeatable process that gives rise to a number of outcomes.

b) An **event** is a collection (or set) of one or more out comes.

c) A **sample space** is the set of all possible outcomes of an experiment.

d) An **impossible event** has probability of **0 (zero)**

e) An **event that is certain** has probability of **1 (one)**

f) As **all events** have probabilities **between impossible (0)** and **certain (1)**, then probabilities are usually written as a **fraction**, a **decimal**, or sometimes a **percentage**.

Q1) In the **real world** where does probability in statistic apply?

A1) It applies in the **money market**, in **weather reporting** and in **quantum theory**.

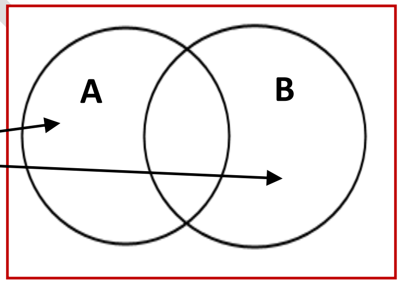
Venn Diagram

In **Venn diagram**:

1- Rectangle represents the **sample space**, and can be labelled **S**, and because it includes all the sample outcomes of an experiment therefore the probability of the sample space is one (1).

P(S) = 1

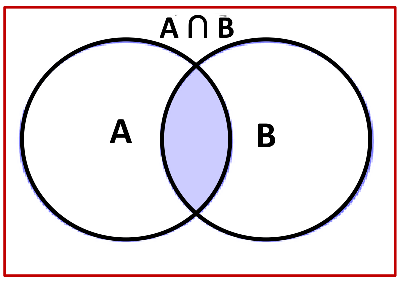
Events of **A & B**



2- Event A ∩ B

A intersection **B**

(A∩B)

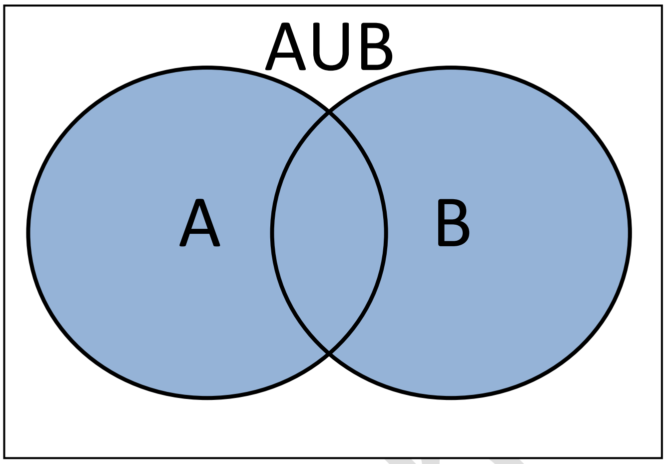


3- Event **A U B**

A union **B**

(AUB)

A or **B**



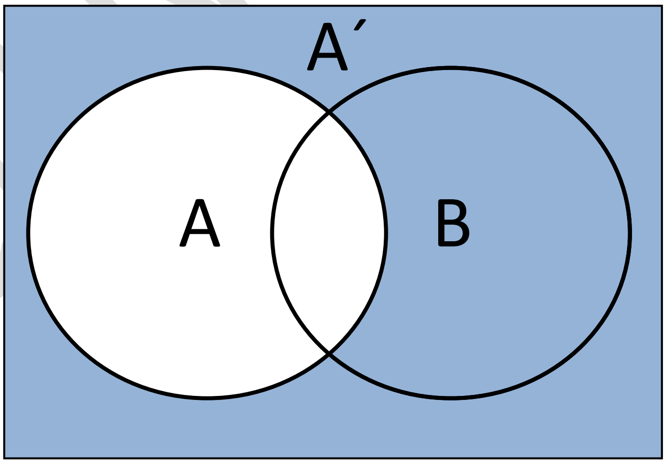
4- Event **A´**

Not **A**

P(A´) = 1 – P(A)

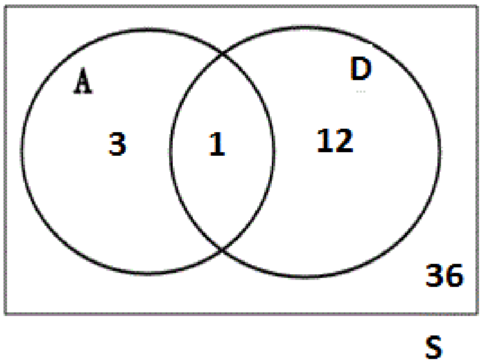
Therefore

P(B´) = 1 – P(B)



Examples

$$P(A\cap D) = \frac{1}{52}$$



$$P'(A\cap D) = 1 - \left(\frac{1}{52}\right)$$

$$= \frac{52-1}{52} = \frac{51}{52}$$

$$P(A\cup D) = \frac{12+1+3}{52} = \frac{4}{13}$$

$$P(A\cup D) = P(A) + P(D) - P(A\cap D)$$

$$= \left(\frac{4}{52} + \frac{13}{52}\right) - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

$$P(A'\cup D') = 1 - P(A\cup D)$$

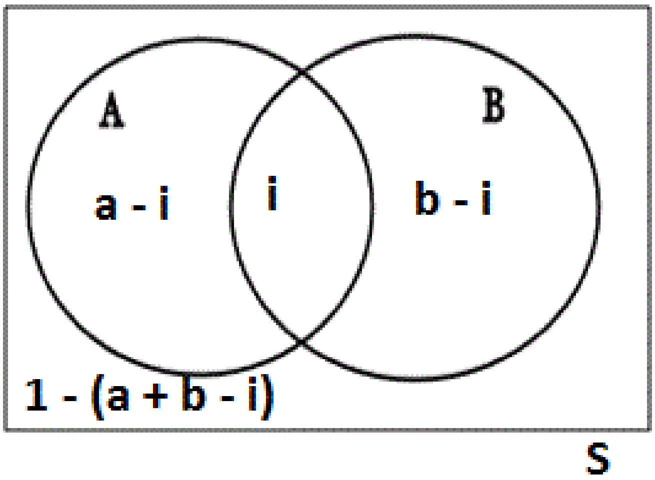
$$= 1 - \frac{4}{13} = \frac{13-4}{13} = \frac{9}{13}$$

$$P(A') = 1 - P(A) = 1 - \frac{4}{52} = \frac{52-4}{52} = \frac{48}{52} = \frac{12}{13}$$

$$P(A'\cap D) = \frac{12}{52} = \frac{3}{13}$$

$$P(A) \text{ only} = P(A) - i = 4 - 1 = 3$$

Using Formulae to Solve Problems



$$P(AUB) = (a - \cancel{i}) + (b - i) + \cancel{i}$$

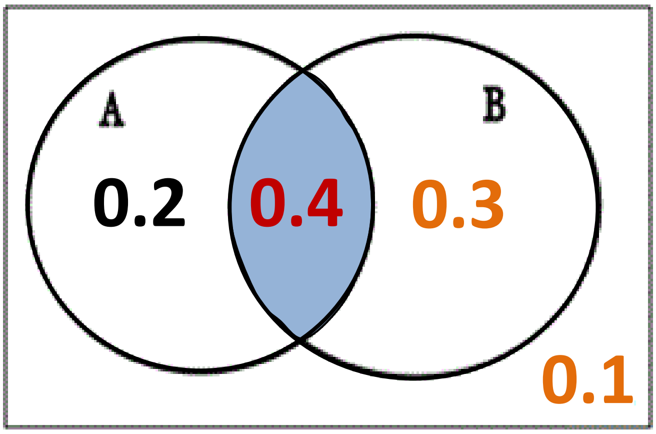
$$= a + b - i$$

Or

$$P(AUB) = P(A) + P(B) - P(A \cap B)$$

To find intersection:

$$P(A \cap B) = P(A) + P(B) - P(AUB)$$



IMPORTANT:

P(A´UB) = not A + rest of B

= (0.1 + 0.3) + 0.4 = 0.8

NOTICE:

If for example we were given that, find: **P(CUD´)**

We need to change it to: **P(D´UC)**

And then, use the above formula.

P(D´UC) = not D + rest of C

Exercise

Q4) On a firing range, a rifleman has two attempts to hit a target.

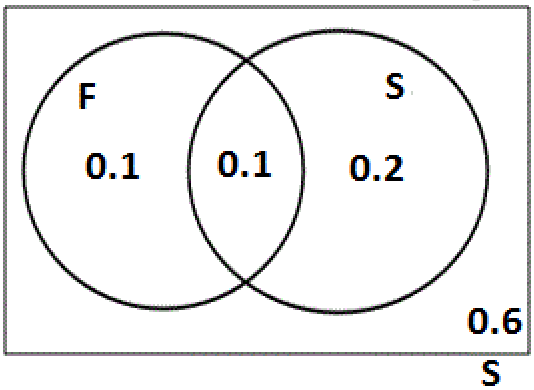
The probability of hitting the target with the first shot is 0.2 and the probability of hitting with the second shot is 0.3.

The probability of hitting the target with both shots is 0.1.

a) Missing the target with both shots,

b) Hitting with the first shot and missing with the second.

A3):



a) **P(FUS)** = 0.2 + 0.3 -0.1 = 0.4

P(S) = 0.3

P(S´) = 1 – 0.3 = 0.7

P(F∩S) = 0.1

$$P(F \cup S) = a + b - i = 0.2 + 0.3 - 0.1 = 0.4$$

$$P(F' \cup S') = 1 - P(F \cup S) = 1 - 0.4 = 0.6$$

b) $P(F \cap S') = P(f) + P(S') - P(F \cup S')$

$$P(F \cup S') = P(S' \cup F)$$

= **not** S + **rest of** F

$$= 0.7 + 0.1$$

$$= 0.8$$

Therefore:

$$P(F \cap S') = P(f) + P(S') - P(F \cup S')$$

$$= 0.2 + 0.7 - 0.8$$

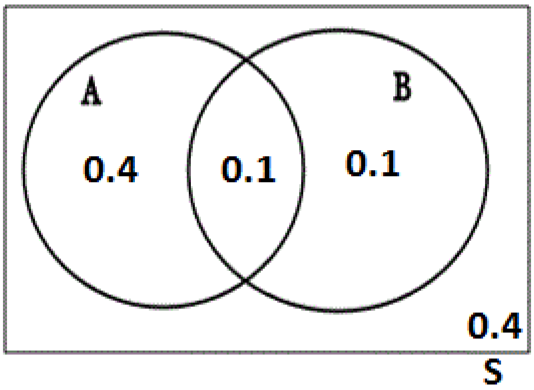
$$= 0.1$$

Q5) A and B are two events and P(A) = 0.5, P(B) = 0.2, and P(A∩B) = 0.1 Find:

a) P(AUB), **b)** P(B´),

c) P(A∩B´), **d)** P(AUB´).

A5):



a) 0.6

b) 0.8

c) P(A∩B´) = P(B´∩A)

P(B´∩A) = P(B´) + P(A) – P(B´UA)

$$= 0.8 + 0.5 - \left(\underbrace{(0.4 + 0.4)}_{\text{Not B}} + \underbrace{0.1}_{\text{Rest of A}}\right)$$

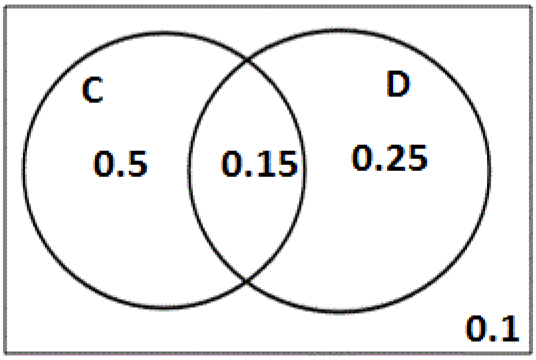
$$= 0.13 - 0.9 = 0.4$$

d) P(AUB´) = P(B´UA) = (0.4 + 0.4) + 0.1

= 0.9

Q6) C and D are two events and $P(D) = 0.4$, $P(C \cap D) = 0.15$ and $P(C' \cap D') = 0.1$. Find

a) $P(C' \cap D)$, **b)** $P(C \cap D')$, **c)** $P(C)$, **d)** $P(C' \cap D')$.



a) $P(C' \cap D) = P(C') + P(D) - P(C' \cup D)$

$= 0.35 + 0.4 - ((0.25 + 0.1) + 0.15)$

Not **C** Rest of **D**

$= 0.75 - 0.5 = \mathbf{0.25}$

b) $P(C \cap D') = P(D' \cap C)$

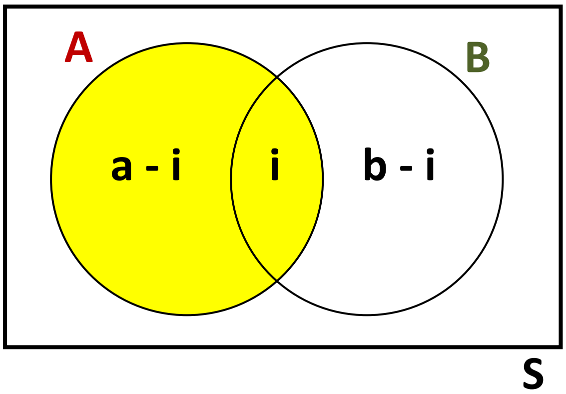
$P(D' \cap C) = P(D') + P(C) - P(D' \cup C)$

$= 0.6 + 0.65 - ((0.5 + 0.1) + 0.15)$

Not **D** Rest of **C**

$= 0.5$

Solving Problems using Conditional Probability



We are looking for the probability of **B** given **A** has already happened, so we rescale the probability of **A** to **1**.

We can think that sample **S** has shrunken to **A**.

Remember:

One condition depends on another.

When you see the word given that, or given, you should use the following formulae:

Probability of B given A:

$$P(B|A) = \frac{i}{a} = \frac{P(B \cap A)}{P(A)}$$

So:

$$P(B \cap A) = P(B|A) \times P(A)$$

Or if we are given **P(A|B)** then:

$$P(A \cap B) = P(A|B) \times P(B)$$

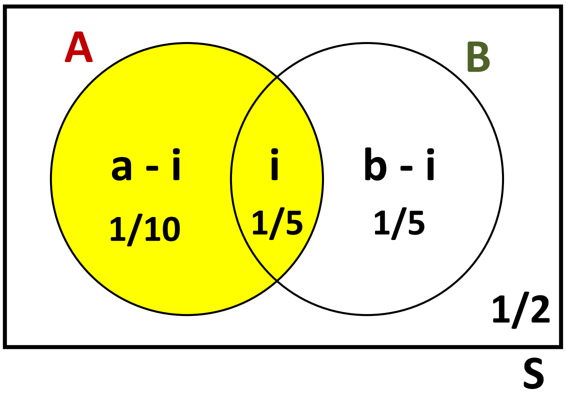
Also:

$$P(A' \cap B) = P(A') + P(B) - P(A' \cup B)$$

But there is a quicker formula for the above:

$$P(A' \cap B) = P(B) - P(A \cap B)$$

Exercise



Find: $P(A \cap B')$

$$P(A \cap B') = P(A) + P(B') - P(A \cup B')$$

$$= \left(\tfrac{1}{10} + \tfrac{1}{5}\right) + \left(\tfrac{1}{10} + \tfrac{1}{2}\right) - \left(\underbrace{\left(\tfrac{1}{10} + \tfrac{1}{2}\right)}_{\text{Not } B} + \underbrace{\tfrac{1}{5}}_{\text{Rest of } A}\right)$$

$$= \tfrac{1}{10}$$

There is a quicker formula for the above:

$$P(A \cap B') = P(A) - P(A \cap B)$$

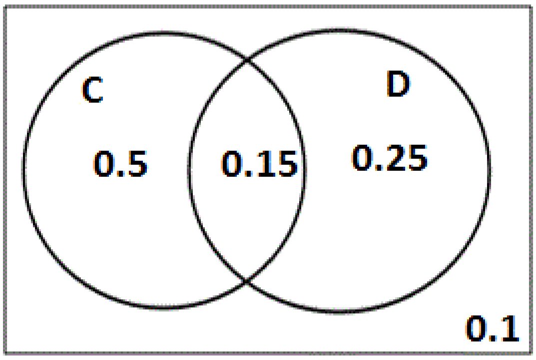
$$= \left(\tfrac{1}{10} + \tfrac{1}{5}\right) - \tfrac{1}{5}$$

$$= \tfrac{1}{10}$$

As you can see we have the same answers.

Q6-again) **C** and **D** are two events and **P(D) = 0.4**, **P(C∩D) = 0.15** and **P(C′∩D′) = 0.1** Find:

a) $P(C' \cap D)$ **b)** $P(C \cap D')$



a) $$P(C' \cap D) = P(C') + P(D) - P(C' \cup D)$$

$$= 0.35 + 0.4 - ((0.25 + 0.1) + 0.15)$$

(0.25 + 0.1): **Not C** ; 0.15: **Rest of D**

$$= 0.75 - 0.5$$

$$= \mathbf{0.25}$$

We have done this exercise before but now we can use a quicker formula to reach to the answer:

$$P(C' \cap D) = P(D) - P(C \cap D)$$

$$= 0.4 - 0.15$$

$$= \mathbf{0.25}$$

As you can see we have the same answers.

b) P(C∩D´) = P(D´∩C)

P(D´∩C) = P(D´) + P(C) – P(D´UC)

= 0.6 + 0.65 – **(**(0.5 + 0.1) + 0.15**)**

(0.5 + 0.1): **Not D**; 0.15: **Rest of C**

= **0.50**

we can use the new formula here too:

P(C∩D´) = P(C) – P(C∩D)

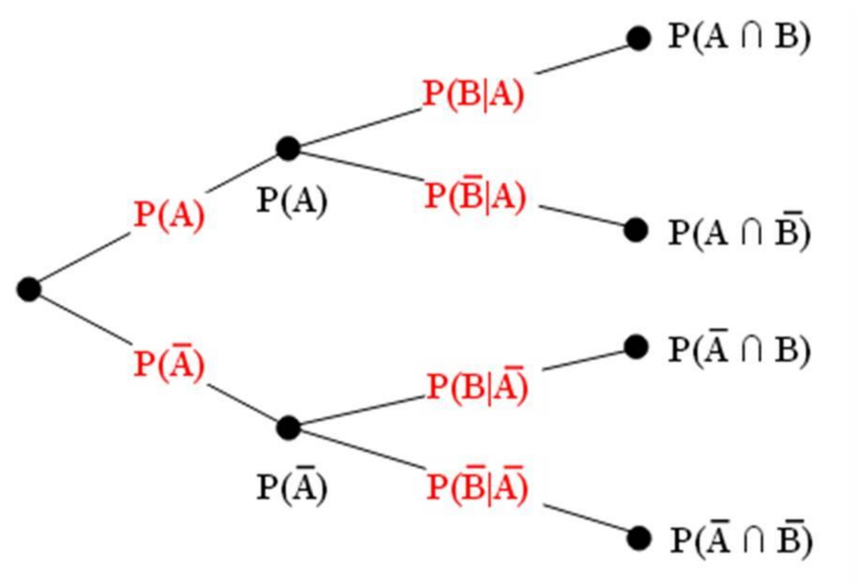
= 0.65 – 0.15

= **0.50**

The same answers in here too.

Conditional Probabilities can be Represented on a Tree Diagram

A **conditional probability** is the probability that an event will occur, when another event is known to occur or to have occurred. If the events are **A** and **B** respectively, this is said to be "the probability of **B given A**". It is commonly denoted by **P(B|A)**.



On a tree diagram, branch probabilities are conditional on the event associated with the parent node.

Remember:

Moving **along** the branches of a tree diagram you **multiply probabilities**, and moving **between** branches you **add probabilities**.

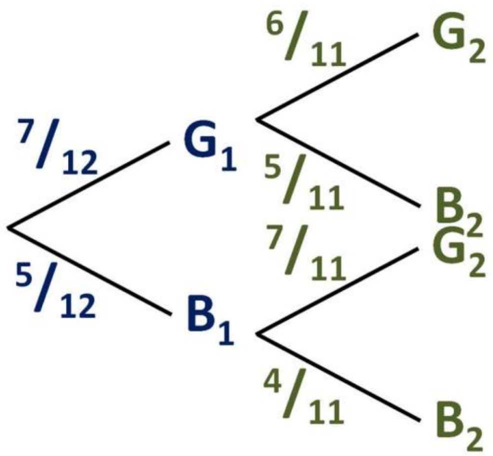
The probabilities on the **second tier of branches** are **conditional** upon the **first tier of branches**.

Exercise

Q) A bag contains **seven green** beads and **five blue** beads. A bead is taken from the bag at random, the colour is recorded and it is **not replaced**. A second bead is then taken from the bag and its colour is recorded. **Find the probability** that **one bead is green <u>and</u> another bead is blue**.

A): First we have **12** beads on the **first tier of branches** with **7 Green** (labelled G_1) on **<u>one</u> branch** and **5 Blue** (labelled B_1) on the **<u>next</u> branch**.

There are only **11** beads on the **second tier of branches** (the bead which was taken was not replaced).



P(one bead is Green and another is Blue)

= P(First Green Then Second Blue) + P(First Blue Then Second Green)

Multiply along the branches

$$= {}^{7}/_{12} \times {}^{5}/_{11} + {}^{5}/_{12} \times {}^{7}/_{11}$$

Add between the branches

$$= {}^{35}/_{66}$$

Q) The turn out of spectators at a motor rally is dependent upon the weather. On a rainy day the probability of a big turnout is 0.4 but if it does not rain, the probability of a big turnout increases to 0.9 the weather forecast gives the probability of 0.75that it will rain on the day of the race.

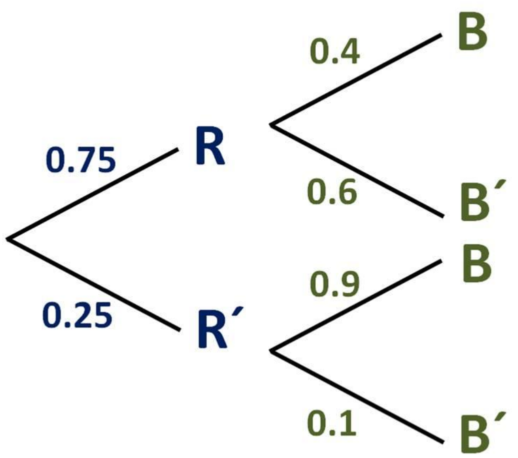
a) Draw a tree diagram to represent this information.

Find the probability that:

b) There is a big turnout and it rains.

c) There is a big turnout.

A -a):



A -b):

$$P(B \cap R) = P(B|R) \times P(R)$$

$$= 0.4 \times 0.75$$

$$= 0.3$$

A -c): In this part we are tolled to find the probability that there is a **big turnout**, and as it dose not specifies that when it rain or when it dose not rain, therefore we have to find **both conditions**.

In **b)** we worked out probability of a **big** turnout and it **rains = 0.3**

Now we work out probability of a **big** turnout and it does **not rain**:

$$P(B \cap R') = P(B|R') \times P(R')$$

$$= 0.25 \times 0.9$$

$$= 0.225$$

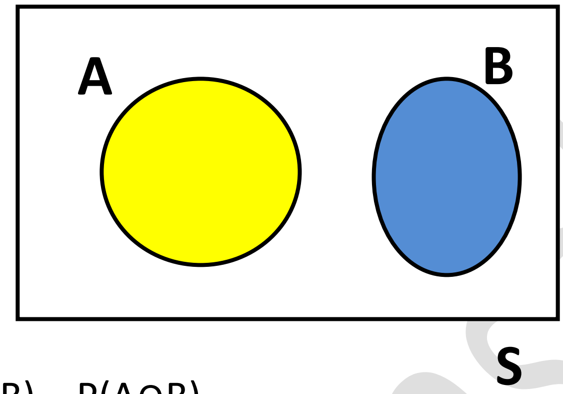
∴ Probability of a **big turnout** is:

$$P(B) = 0.3 + 0.225$$

$$= 0.525$$

Mutually Exclusive Events

When events have **no outcomes in common**, they are **mutually exclusive**.



$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

AS:

$P(A \cap B) = 0$

∴

$$\mathbf{P(A \cup B) = P(A) + P(B)}$$

There is no intersection between **A** and **B** therefore **Not A** overlaps with the whole of **A** (all of **A** is the intersection between **A** and **Not A**).

∴

$$\mathbf{P(A \cap B') = P(A)}$$

The intersection of **A´** and B´ is S without A and B.

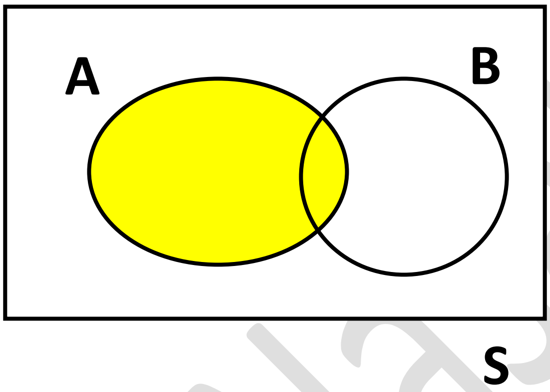
As $P(S) = 1$

∴

$$\mathbf{P(A' \cap B') = 1 - P(A \cup B)}$$

Independent Events

If **P(A|B)** is equal to the **P(A)**. they are said to be **independent**. For example, if a coin is flipped twice, "the outcome of the second flip" is **independent** of "the outcome of the first flip".



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

When one event has no effect on another they are independent, therefore **A** and **B** are independent, and probability of

A given **B** is still = **P(A)**

$$P(A|B) = P(A)$$

We have: $P(A \cap B) = P(A|B) \times P(B)$

We substitute $P(A|B)$ in the above formulae by $P(A)$ and we get:

$$P(A \cap B) = P(A) \times P(B)$$

$$P(A' \cap B') = P(A') \times P(B') \text{ or } P(A' \cap B') = 1 - P(A \cup B)$$

$$P(A \cap B') = P(A) \times P(B') \text{ or } P(A \cap B') = P(A) - P(A \cap B)$$